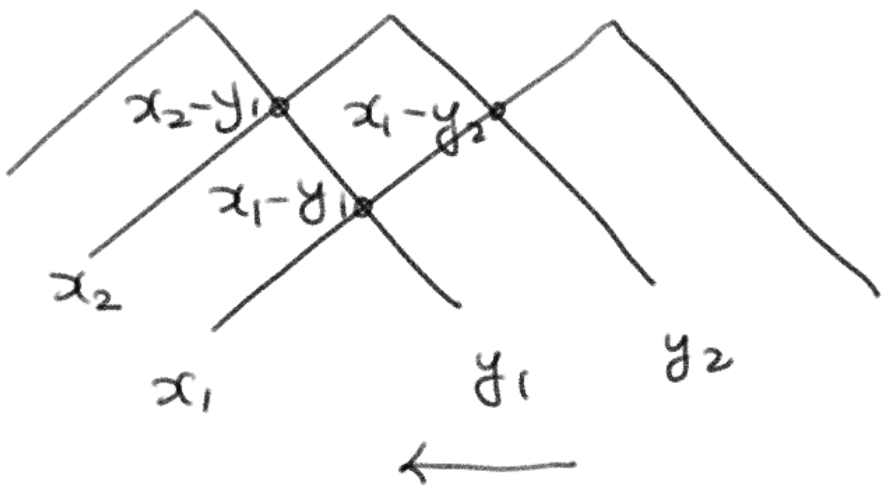
__proof__ (lemma)

$$\partial_i S = S u_i \iff \frac{1}{x_i - x_{i+1}}(S - s_i S) = S u_i$$
$$\iff S - s_i S = S(x_i - x_{i+1}) u_i$$
$$\iff S(1 + (x_{i+1} - x_i) u_i) = s_i S$$
$$\iff S h_i(x_{i+1} - x_i) = s_i S$$

__Ex__ (n=3)



$$S = h_2(x_1 - y_2) h_1(x_1 - y_1) h_2(x_2 - y_2)$$

__i=2__ $h_2(x_1 - y_2) h_1(x_1 - y_1) h_2(x_2 - y_2) h_2(x_3 - x_2) =$

$= h_2(x_1 - y_2) h_1(x_1 - y_1) h_2(x_3 - y_2)$ YB

Generally, we can understand the proof from wiring diagram and YB relations.

Cauchy Type Identities

(1) Cauchy Formula for Schur Polynomials S_λ (fix n)

$$\sum_{\lambda=(\lambda_1,\cdots,\lambda_n)} S_\lambda(x_1,\cdots,x_n)S_\lambda(y_1,\cdots,y_n) = \prod_{\substack{(i,j)\\ i,j\in[n]}} \frac{1}{1-x_iy_j}$$

(2) Dual Cauchy for S_λ (fix m,n)

$$\sum_{\lambda\subset m\times n \text{ rectangle}} S_\lambda(x_1,\cdots,x_m)S_{\lambda'}(y_1,\cdots,y_n) = \prod_{\substack{i\in[m]\\ j\in[n]}}(1+x_iy_j)$$

conjugate partition

(3) Cauchy for Schubert Polynomial

$$\sum_{w\in S_n} \mathfrak{S}_w(x_1,\cdots,x_{n-1})\mathfrak{S}_{w^{-1}w_0}(y_1,\cdots,y_{n-1}) = \prod_{\substack{(i,j)\\ i+j\leq n}}(x_i+y_j)$$

Common generalization of (1) & (2) & (3)

Thm $w \in S_n$ then

$$\mathfrak{S}_w(x_1,\dots,x_{n-1}; -y_1, -y_2, \dots, -y_{n-1}) =$$

$$= \sum_{\substack{u,v \in S_n \\ w = u \cdot v \\ \ell(w) = \ell(u) + \ell(v)}} \mathfrak{S}_u(x_1,\dots,x_{n-1})\, \mathfrak{S}_v(y_1,\dots,y_{n-1})$$